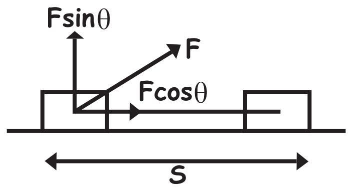
WORK ENERGY & POWER

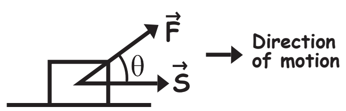
WORK
Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force

WORK DONE BY CONSTANT FORCE

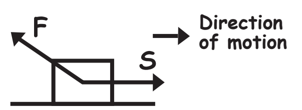


$W = F\cos\theta \times S$
$= FS\cos\theta = \vec{F}.\vec{S}$

NATURE OF WORK DONE
1) Positive work ($0° \le \theta \le 90°$)



2) Negative work ($90° \le \theta \le 180°$)



2) Zero work

Work done becomes 0 for three conditions

1. Force is perpendicular to displacement
2. if there is no displacement
3. if there is no force acting on the body

WORK DONE BY VARIABLE FORCE
$dW = \vec{F}.d\vec{s}$

$W = \int \vec{F}.d\vec{s} = \int Fds\cos\theta$

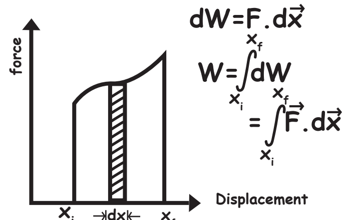
in terms of rectagular components

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$d\vec{s} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

$W = \int F_x dx + \int F_y dy + \int F_z dz$

Graphical representation of work done



$dW = F.d\vec{x}$

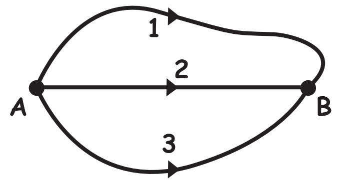
$W = \int_{x_i}^{x_f} dW$

$= \int_{x_i}^{x_f} \vec{F}.d\vec{x}$

W= Area under curve

WORK DONE BY CONSERVATIVE & NON CONSERVATIVE FORCE
Conservative: work done doesnot depend on path followed

Non-conservative: work depends on the path followed



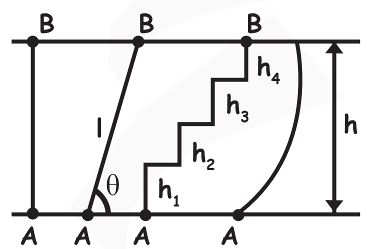
- $W_{A\to B}$(Path 1)$=W_{A\to B}$(Path 2)$=W_{A\to B}$(Path 3)

  (for conservative force)
- $W_{A\to B}$(Path 1)$\neq W_{A\to B}$(Path 2)$\neq W_{A\to B}$(Path 3)

  (for non conservative force)

Note :
Work done for a complete cycle by a conservative force is zero

WORK DONE BY DIFFERENT FORCES



$W_1 = mgh = mgh$

$W_2 = mg \times l\sin\theta = mg \times l \times \frac{h}{l}$

$= mgh$

$W_3 = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4$

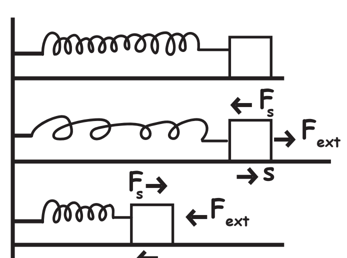
work done by static friction $\rightarrow$ 0

work done by kinetic friction $\rightarrow$ -ve

$W_{fk} = \vec{f_k}.\vec{S} = f_k\,S\cos 180\,,\ = -f_k S$

work done by spring force

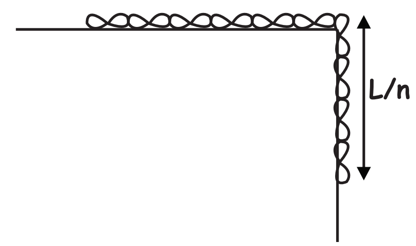
magnitude of spring force=-kx



$W_s = \int \vec{F_s}.d\vec{s} = -\int_{x_i}^{x_f} kx\,dx$

$= -\frac{1}{2}K\left(x_f^2 - x_i^2\right)$

WORK DONE IN PULLING THE CHAIN



L $\rightarrow$ Total length

$(1/n)^{th}$ Part of length hanging

M $\rightarrow$ Mass of chain

Work done in pulling, the hanging portion on the table $W = \frac{MgL}{2n^2}$

ENERGY
- Capacity of doing work
- Scalar quantity
- Dimension ML^2T^{-2}

Relation between different units

$1eV = 1.6\times10^{-19}$Joules

$1kWh = 3.6\times10^6$Joules

1calorie=4.18Joules

1 Joule$=10^7$erg

Kinetic Energy
- Energy possessed by virtue of motion Expression

  $K.E = \frac{1}{2}mv^2$
- Always positive
- Depends on frame of reference

Work Energy Theorm

Change in kinetic Energy

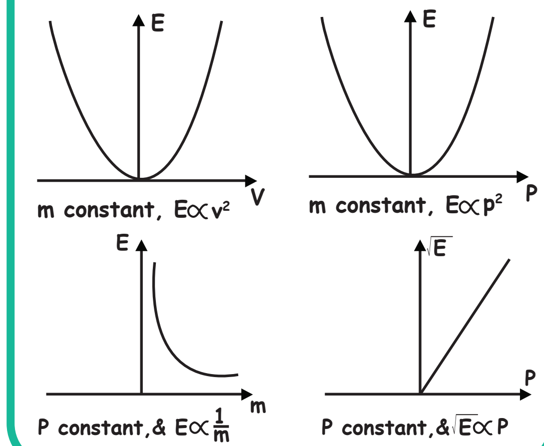
$\Delta K.E = \frac{1}{2}mv^2$

Change in kinetic energy of a body is equal to work done on the body

$K_2 - K_1 = -\int \vec{F}.d\vec{r}$

RELATION OF KINETIC ENERGY WITH OTHER Quantities
Linear momentum:- $P = \sqrt{2m\,E}$

Variation of graph of kinetic Energy



POTENTIAL ENERGY
- Defined only for conservative force
- Energy possessd by a body by virtue of its position
- Can either be positive, negative or zero according to point of reference
- Body always move from higher potential to lower potential

Identifying forces with potential energy

1) Attractive force:-
On increasing x, if U increases

$\frac{dU}{dx}$=positive

(BC portion of graph)

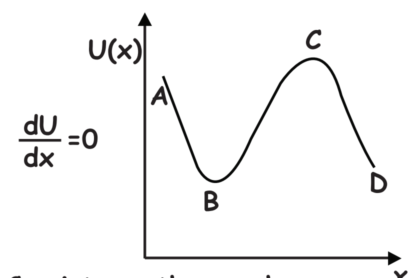
2) Repulsive force:-
On increasing x, if U decreases

$\frac{dU}{dx}$=negative

(AB portion of graph)

3) Zero force:-
On increasing x, if U doesnot change

$\frac{dU}{dx}=0$

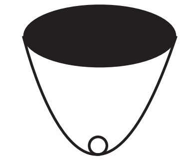


B,C points on the graph

Types of Potential Energy
- Elastic Potential Energy
- Electric Potential Energy
- Gravitational Potential Energy

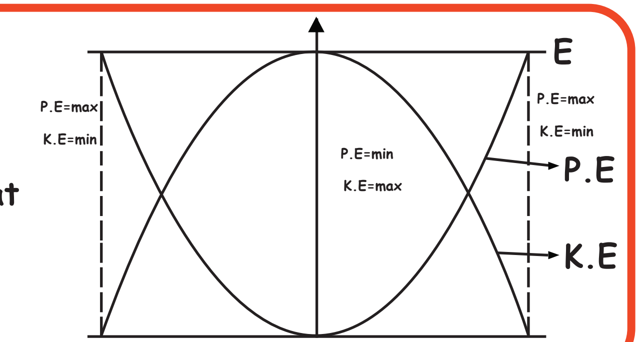
Types of equilibrium

If net force acting on a particle is zero it is said to be in equillibrium

STABLE	UNSTABLE	NUETRAL
If particle displaced from equillibrium position force acting will try to bring back to the initial position	• If particle displaced from equillibrium position force acting on it tries to displace further away from equillibrium position	• If particle is slightly displaced from equillibrium, then it doesn,t experience a force or continues in equillibrium
Potential energy is minimum	• Potential energy is maximum	• Potential energy is constant
$F = \frac{-dU}{dx} = 0$	• $F = \frac{-dU}{dx} = 0$	• $F = \frac{-dU}{dx} = 0$
$\frac{d^2U}{dx^2}$=positive	• $\frac{d^2U}{dx^2}$=negative	• $\frac{d^2U}{dx^2}=0$
		

CONSERVATION OF ENERGY
For an isolated system for a body in presence of conservative forces, the sum of kinetic and potential energies at any point remains constant throughout the motion

K.E+P.E=constant

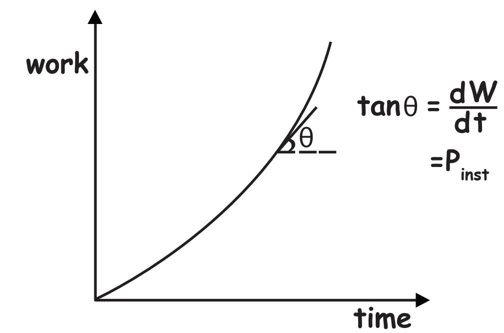


POWER
- Rate at which body can do work
- Average power$(P_{av}) = \frac{\Delta W}{\Delta t}$
- Instantaneous power$(P_{inst}) = \frac{dW}{dt}$

$= \frac{\vec{F}.d\vec{s}}{dt} = \vec{F}.\vec{v}$

Relation between units:
- 1 watt=1joule/sec $=10^7$ erg/sec
- 1 HP=746watt, 1MW $=10^6$ watt
- 1 KW$=10^3$ watt
- If work done by two bodies is same then power $\propto \frac{1}{\text{time}}$
- Unit of power multiplied by time always give work

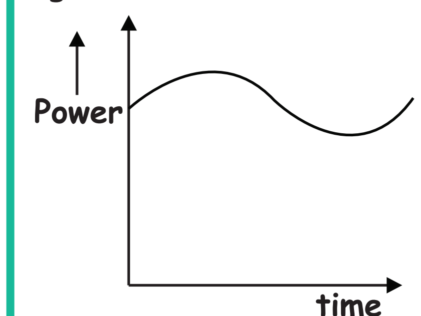
  1 KWh$=3.6\times10^6$ Joules
- Slope of work-time curve gives instantaneous power



$\tan\theta = \frac{dW}{dt}$

$= P_{inst}$

- Area under power time graph gives work done



$P = \frac{dW}{dt}$

$\Rightarrow dW = Pdt$

$W = \int Pdt$

W=Area under P-t graph

WORK ENERGY & POWER

Position and velocity in terms of power:-

1) Velocity, $V=\left[\frac{2Pt}{m}\right]^{1/2}$

2) Position, $S=\left[\frac{8P}{9m}\right]^{1/2}t^{3/2}$

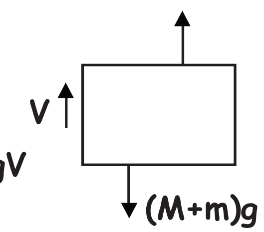
Power delivered by an elevator

a=0, T=(M+m)g

$\vec{P}=\vec{T}.\vec{V}$

$=TV$

Power, $P=(M+m)gV$



Power of a water drawing pump

- Power, $P=\frac{dW}{dt}=\frac{dm}{dt}\left[gh+\frac{V^2}{2}\right]$
- h=height of water level

$\frac{dm}{dt} \Rightarrow$ mass flow rate of pump

V → velocity of the water outlet

- Power required to just lift water, V=0

$P=gh\left(\frac{dm}{dt}\right)$

Efficiency of pump

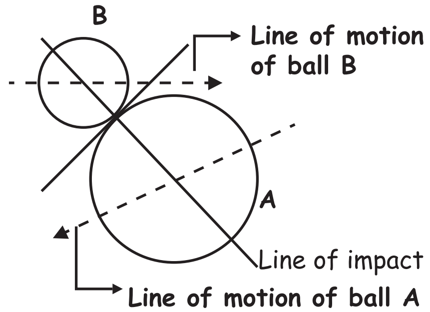
$$\mu=\frac{\text{Output Power}}{\text{Input Power}}$$

COLLISION

Event in which impulsive force acts between two or more bodies which result in change of their velocities.

Line of impact

Line passing through common normal to surfaces in contact during impact



Coefficient of restitution (e)

$$e=\frac{\text{Velocity of separation along the line of impact}}{\text{Velocity of approach along the line of impact}}$$

$$=\frac{\text{Relative velocity after collision along the line of impact}}{\text{Relative velocity before collision along the line of impact}}$$

Conditions

1. For elastic collision: e=1
2. For inelastic collision: e<1
3. For perfectly elastic collision: e=0

Classification

- Based on conservation of kinetic energy
  - Perfectly elastic
  - Inelastic
  - Perfectly inelastic
- Based on direction of colliding bodies
  - Oblique
  - Head on/one dimensional

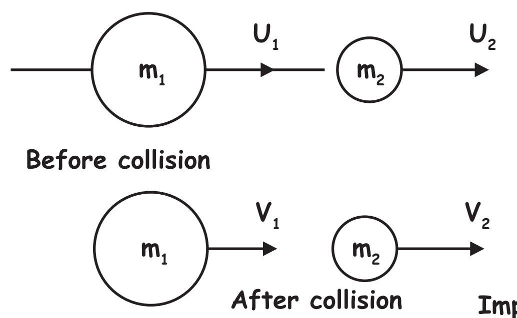
Perfectly elastic collision

K.E before and after collision is same

Inelastic collision

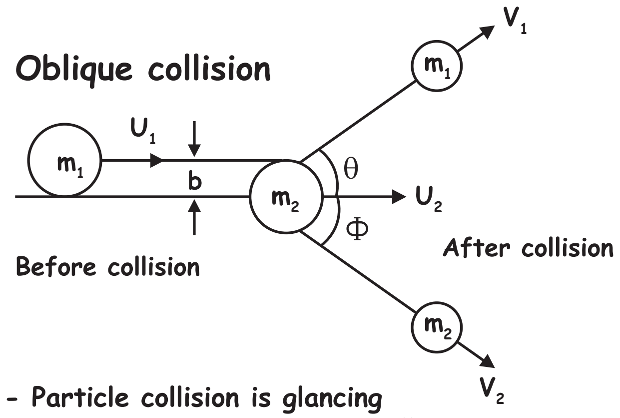
K.E after collision is not equal to K.E before collision then it is said to be inelastic collision

Head on collision / One dimensional collision



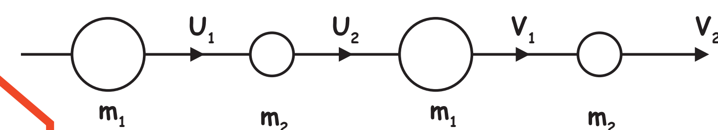
Impact parameter b=0

Oblique collision



- Particle collision is glancing
- Direction of motion after collision are not along initial line of motion
- If they collide in same plane , collision is 2 dimensional otherwise 3 dimensional
- Impact parameter $0<b<(r_1+r_2)$ r_1,r_2 are radii of colliding bodies

Perfectly elastic Head on collision



Velocity after collision

$$V_1=U_1\left[\frac{m_1-m_2}{m_1+m_2}\right]+\frac{2m_2U_2}{m_1+m_2}$$

$$V_2=U_2\left[\frac{m_2-m_1}{m_1+m_2}\right]+\frac{2m_1U_1}{m_1+m_2}$$

Special cases:

1) Projectile and target having same mass $m_1=m_2$ $v_1=u_2, v_2=u_1$, the velocities get interchanged

2) If massive projectile collide with a light target ie $m_1>>m_2$ $v_1=u_1, v_2=-u_2+2u_1$

3) If the light projectile collides with a very heavy target, $m_1<<m_2$ $v_1=-u_1+2u_2, v_2=u_2$

Energy transfer from projectile to target

1) Fractional decrease in kinetic energy (If target is at rest)

$$\frac{\Delta K}{K}=\frac{4m_1m_2}{(m_1-m_2)^2+4m_1m_2}$$

Greater the difference in masses, less will be transfer of K.E and vice versa

If $m_2=nm_1$ $\frac{\Delta K}{K}=\frac{4n}{(1+n)^2}$

Inelastic collision

$$e=\frac{V_2-V_1}{U_1-U_2}\quad\frac{\text{Relative velocity of seperation}}{\text{Relative velocity of approach}}$$

Velocity after collision $V_1=\frac{(1+e)\,m_2U_2}{m_1+m_2}+\frac{(m_1-em_2)U_1}{m_1+m_2}$

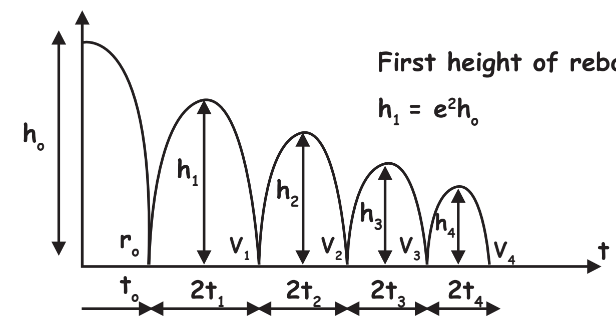
$V_2=\frac{(1+e)\,m_1U_1}{m_1+m_2}+\frac{(m_2-em_1)U_2}{m_1+m_2}$

Ratio of velocities

$\frac{V_1}{V_2}=\frac{1-e}{1+e}$

Loss in kinetic energy $\Delta K=\frac{1}{2}\left[\frac{m_1m_2}{m_1+m_2}\right](1-e^2)\,(U_1-U_2)^2$

Rebounding of ball



First height of rebound

$h_1=e^2h_0$

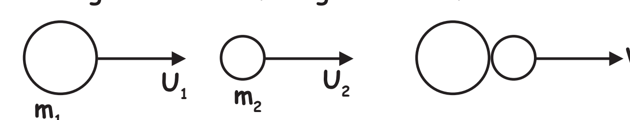
Total distance travelled by the ball before it stops bouncing

$$H=h_0\left[\frac{1+e^2}{1-e^2}\right]$$

Total time taken by the by to stop bouncing $T=\left(\frac{1+e}{1-e}\right)\frac{2h_0}{g}$

Perfectly inelastic collision

colliding bodies are moving in the same direction



$V=\frac{m_1U_1+m_2U_2}{m_1+m_2}$

Loss in kinetic energy $\Delta k=\frac{1}{2}\frac{m_1m_2}{m_1+m_2}(U_1-U_2)^2$

Colliding bodies are moving in the opposite direction

$V=\frac{m_1U_1-m_2U_2}{m_1+m_2}$ Change in kinetic enargy $\Delta K=\frac{1}{2}\frac{m_1m_2}{m_1+m_2}(U_1+U_2)^2$

Collision in two dimension

If the initial velocities of two colliding bodies are not along the line of impact, then the collision is said to be oblique collision or collision in two dimension.